DIFFUSION BURNING IN A SYSTEM OF TWO-DIMENSIONAL TURBULENT JETS

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Diffusion burning of nonpremixed gases in a system of two-dimensional turbulent jets is examined. The problem is solved with the aid of a numerical method. Analytical results obtained for a hydrogen jet in a jet of air are given in the form of graphs.

Heat- and mass-transfer processes in turbulent jets in the presence of chemical reactions are of considerable scientific and practical interest.

Unfortunately, the study of such processes is associated with great difficulties. These difficulties are caused, on the one hand, by our insufficient knowledge of the kinetics of chemical processes of practical interest and, on the other hand, by the complete absence of a rationally substantiated theory of turbulent transfer processes. Consequently, all existing methods of calculating jet flows in the presence of heat- and mass-transfer and chemical processes [1–5] are based on the extension of classical semiempirical theories of free turbulence to include these more complex flows.

The present work, which in this sense is no exception among the papers cited, is dedicated to the study of motions in an infinite system of two-dimensional turbulent jets in the presence of diffusion burning.

We examine the flow which forms in the mixing zone of an infinite system of turbulent jets [10] expelled from two-dimensional nozzles of a width of $2a$ (Fig. 1). The walls between the jets are assumed infinitely thin. Assume that one of the nozzles (1) expels an oxidizer while the neighboring nozzle (2) expels a fuel. The conditions at the nozzle outlet are taken as homogeneous, such that in the entire right half-plane, there occurs a periodic flow—with period $4a$—that is symmetrical with respect to the axis of an arbitrary jet. It is therefore sufficient to study the channel between any two neighboring central axes—for example, between the axes $y = 0$ and $y = 2a$, which constitute streamlines. The axis $y = 0$ will be taken as the zero streamline.

For a turbulent Lewis number of unity, the basic equations which describe the mean stationary motion and the transport processes can be written in the form

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\rho\varepsilon \frac{\partial u}{\partial y}\right);$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0;$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{1}{\mathrm{Pr}} \frac{\partial}{\partial y}\left(\rho\varepsilon \frac{\partial H}{\partial y}\right) -$$

$$-\left(\frac{1}{\mathrm{Pr}} - 1\right)\frac{\partial}{\partial y}\left[\rho\varepsilon \frac{\partial\left(\frac{u^2}{2}\right)}{\partial u}\right];$$

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = \frac{1}{\mathrm{Sc}} \frac{\partial}{\partial y}\left(\rho\varepsilon \frac{\partial c_i}{\partial y}\right) + w_i;$$

$$p = \rho \frac{R_0}{m} T; \quad m = \left(\sum_{i=1}^{N} \frac{c_i}{m_i}\right)^{-1}, \tag{1}$$

where

$$H = c_p T + \sum_{i=1}^{N} c_i h_i^* + \frac{u^2}{2}. \tag{2}$$

The system of equations (1) must be solved for the following boundary conditions:

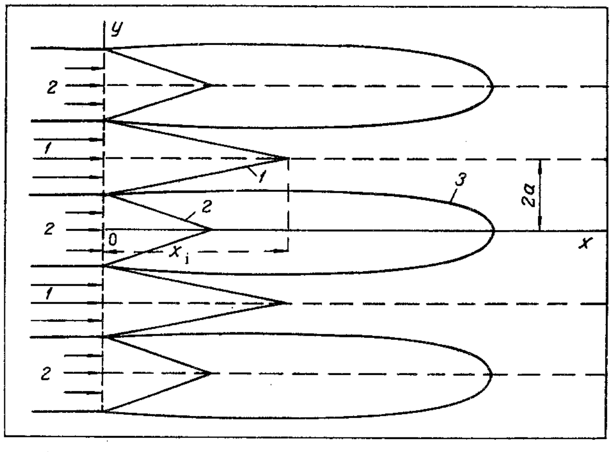

Fig. 1. Flow in an infinite system of two-dimensional jets in the presence of a diffusion flame front: 1) $y_1(x)$, 2) $y_2(x)$, 3) $y_*(x)$.

in the initial cross section for x = 0

$$u = u_-; \quad H = H_-; \quad (c_i) = (c_i)_-; \quad \rho = \rho_-, \quad 0 \leqslant y < a;$$

$$u = u_+; \quad H = H_+; \quad (c_i) = (c_i)_+; \quad \rho = \rho_+, \quad a < y \leqslant 2a, \tag{3}$$

in any cross section y = 0; $2a$

$$\frac{\partial u}{\partial y} = v = \frac{\partial H}{\partial y} = \frac{\partial c_i}{\partial y} = 0. \tag{4}$$

In addition to (3) and (4), it is necessary to satisfy the following integral conditions of mass conservation, total momentum conservation, and enthalpy of the mixture conservation:

$$\int\limits_0^{2a} \rho u dy = M_0 = \text{const} = a(\rho_+ u_+ + \rho_- u_-), \tag{5}$$

$$\int\limits_0^{2a} (p + \rho u^2)\, dy = I_0 = \text{const}, \tag{6}$$

$$\int\limits_0^{2a} H \rho u dy = H_0 = \text{const}. \tag{7}$$

In the following, for convenience, we treat a gas mixture in the mixing zone as a four-component mixture consisting of the oxidizer (subscript 1), the fuel (subscript 2), the product (subscript 3), and an inert gas (subscript 4).

With the aid of a stoichiometric relation between the amounts of burned-up substances

$$w_k \nu_3 m_3 + w_3 \nu_k m_k = 0, \tag{8}$$

where ν_k is the number of molecules of the k-th component participating in the reaction, from the fourth equation in system (1) it may be found that

$$\rho u \frac{\partial C_k}{\partial x} + \rho v \frac{\partial C_k}{\partial y} = \frac{1}{\text{Sc}} \frac{\partial}{\partial y}\left(\rho\varepsilon \frac{\partial C_k}{\partial y}\right), \tag{9}$$

where

$$C_k = c_k \nu_3 m_3 + c_3 \nu_k m_k, \quad k = 1; \quad 2. \tag{10}$$

It should be noted that w_3 and w_k are of different sign.

For incompressible fluid jets, according to the new Prandtl theory, the kinematic coefficient of turbulent viscosity, ε, has the form

$$\varepsilon(x) = \varkappa\, b(x)(u_d - u_b). \tag{11}$$

We assume that formula (11) holds also in our case, where $b(x) = y_1(x) - y_2(x)$ for $x \leq x_i$ and $b(x) = 2a = \text{const}$ for $x > x_i$ **(Fig. 1).**

By introducing a stream function ψ, defined by the equalities

$$\rho u = \frac{\partial \psi}{\partial y}, \quad \rho v = -\frac{\partial \psi}{\partial y}, \tag{12}$$

we satisfy identically the continuity equation.

Further, in the initial system of equations (1), we transform to the dimensionless variables

$$u' = \frac{u}{u_+}, \quad \rho' = \frac{\rho}{\rho_+}, \quad H' = \frac{H}{H_+}, \quad T' = \frac{T}{T_+},$$

$$p' = \frac{p}{\rho_+ u_+^2}, \quad y' = \frac{y}{2a}, \quad x' = \frac{x}{2a}$$

and from the variables x', y', to the generalized Mises variables [6]

$$\xi' = \frac{4\varkappa}{(1+\rho'_- u'_-)^2} \int\limits_0^{x'} b' \,[u'_d - u'_b]\, dx'. \tag{13}$$

After simple transformations, we get for (1) the following system:

$$\frac{\partial u'}{\partial \xi'} = -\frac{1}{\rho' u'} \frac{\partial p'}{\partial \xi'} + \frac{\partial}{\partial \psi'}\left[\rho'^2 u' \frac{\partial u'}{\partial \psi'}\right],$$

$$\frac{\partial H'}{\partial \xi'} = \frac{1}{\text{Pr}} \frac{\partial}{\partial \psi'}\left[\rho'^2 u' \frac{\partial H'}{\partial \psi'}\right] -$$

$$-\frac{u_+^2}{H_+}\left[\frac{1}{\text{Pr}} - 1\right] \frac{\partial}{\partial \psi'}\left[\rho'^2 u'^2 \frac{\partial u'}{\partial \psi'}\right],$$

$$\frac{\partial C}{\partial \xi'} = \frac{1}{\text{Sc}} \frac{\partial}{\partial \psi'}\left[\rho'^2 u' \frac{\partial C}{\partial \psi'}\right],$$

$$p' = \frac{R_0 T_+}{u_+^2} \frac{\rho' T'}{m}. \tag{14}$$

Conditions (3) and (4) reduce, respectively, to the form:

$$u' = u'_-, \ H' = H'_-, \ \rho' = \rho'_-,$$

$$C = 1 \quad 0 \leqslant \psi' < \frac{\rho'_- u'_-}{1+\rho'_- u'_-},$$

$$u' = 1, \ H' = 1, \ \rho' = 1, \ C = 0$$

$$\frac{\rho'_- u'_-}{1+\rho'_- u'_-} < \psi' \leqslant 1, \tag{15}$$

$$\frac{\partial u'}{\partial \psi'} = \frac{\partial H'}{\partial \psi'} = \frac{\partial C}{\partial \psi'} = 0 \text{ for } \psi' = 0; \ 1. \tag{16}$$

In system (14), we have introduced the notation

$$C = \frac{(C_k)_+ - C_k}{(C_k)_+ - (C_k)_-}. \tag{17}$$

From formula (10), it follows that in the last relation in (17)

$$(C_1)_+ = (c_1)_+ \nu_3 m_3; \quad (C_1)_- = 0;$$

$$(C_2)_+ = 0; \quad (C_2)_- = (c_2)_- \nu_3 m_3.$$

It can be readily seen that the equation for determining the inert gas concentration ($w_4 = 0$) reduces to the form of the third equation in system (14). The boundary conditions for the complex $((c_4)_+ - c_4)/((c_4)_+ - (c_4)_-)$ are the same as the corresponding conditions (15) and (16) for the complex denoted by $C(\xi', \psi')$ (equality (17)). Consequently

$$\frac{(c_4)_+ - c_4}{(c_4)_+ - (c_4)_-} = C(\xi', \psi'); \tag{18}$$

hence

$$c_4 = (c_4)_+ - [(c_4)_+ - (c_4)_-]\, C(\xi', \psi'). \tag{19}$$

We assume further that we are dealing with diffusion burning, i. e., that the chemical reaction takes place solely on an infinitely thin surface (flame front), where the stoichiometric relations (8) are fulfilled and where the entire amount of fuel and oxidizer supplied burns up. Hence, on one side of the flame front, we have a zero oxidizer concentration, and on its other side a zero fuel concentration.

In the area between the flame front and the channel boundary ($0 \leq \psi' \leq \psi'_*$) $c_1 = 0$, while from relations (10) and (17) it may be found that

$$c_2 = \left[(c_2)_- + \frac{\nu_2 m_2}{\nu_1 m_1} (c_1)_+ \right] C(\xi', \psi') - \frac{\nu_2 m_2}{\nu_1 m_1} (C_1)_+, \quad (20)$$

$$c_3 = \frac{\nu_3 m_3}{\nu_1 m_1} (c_1)_+ [1 - C(\xi', \psi')]. \quad (21)$$

In the area between the flame front and the other channel boundary ($\psi'_* \leq \psi' \leq 1$), the fuel concentration is zero ($c_2 = 0$). From relations (10) and (17), the oxidizer and reaction-product concentrations are determined as

$$c_1 = (c_1)_+ - \left[(c_1)_+ + \frac{\nu_1 m_1}{\nu_2 m_2} (c_2)_- \right] C(\xi', \psi'), \quad (22)$$

$$c_3 = \frac{\nu_3 m_3}{\nu_2 m_2} (c_2)_- C(\xi', \psi'). \quad (23)$$

At the flame-front surface, the fuel and oxidizer concentrations are zero. Making use of this condition, we find that at the flame front

$$(C_1)_* = (c_3)_* \nu_1 m_1; \quad (C_2)_* = (c_3)_* \nu_2 m_2. \quad (24)$$

With the aid of these relations, from (17), we determine the value of C at the flame front:

$$C_* = \frac{\nu_2 m_2 (c_1)_+}{\nu_2 m_2 (c_1)_+ + \nu_1 m_1 (c_2)_-}. \quad (25)$$

Points on the plane (ξ', ψ') at which the condition $C(\xi', \psi') = C_*$, is fulfilled form the surface of the flame front. It should be noted that for given fuel and oxidizer parameters, the value of C_* is defined solely by the initial fuel and oxidizer concentrations.

From the equation of state, a relation for determining the density can be obtained in the following form:

$$\rho' = \frac{u_+^2}{R_0 T_+} \frac{mp'}{T'}, \quad (26)$$

where T' is determined from the expression for the enthalpy (2)

$$T' = \frac{(c_p)_+}{c_p} \left[H' - \sum_{i=1}^{N} c_i h_i^* - \frac{u_+^2}{H_+} \frac{u'^2}{2} \right]. \quad (27)$$

The system of equations (14) will be solved by the network method. To this end, we divide the plane (ξ', ψ') into rectangles by means of the straight lines $\xi'_i = ih$, $\psi'_j = jl$ ($i = 0, 1, 2, \ldots$, $j = 0, 1, 2, \ldots, n$). For each internal point (ξ'_i, ψ'_i), Eq. (14) will be replaced by a system of finite-difference equations [7]:

$$\frac{u'_{i,j} - u'_{i-1,j}}{h} = - \frac{1}{(\rho' u')_{i,j}} \frac{p'_i - p'_{i-1}}{h} + \frac{1}{l} \times$$
$$\times \left[\Lambda_{i,j+1} \frac{u'_{i,j+1} - u'_{i,j}}{l} - \Lambda_{i,j} \frac{u'_{i,j} - u'_{i,j-1}}{l} \right]; \quad (28)$$

$$\frac{H'_{i,j} - H'_{i-1,j}}{h} = \frac{1}{\mathrm{Pr}} \frac{1}{l} \left[\Lambda_{i,j+1} \frac{H'_{i,j+1} - H'_{i,j}}{l} - \right.$$
$$\left. - \Lambda_{i,j} \frac{H'_{i,j} - H'_{i,j-1}}{l} \right] -$$
$$- \frac{u_+^2}{H_+} \left(\frac{1}{\mathrm{Pr}} - 1 \right) \frac{1}{l} \left[\Lambda_{i,j+1} u'_{i,j+1} \frac{u'_{i,j+1} - u'_{i,j}}{l} - \right.$$
$$\left. - \Lambda_{i,j} u'_{i,j} \frac{u'_{i,j} - u'_{i,j-1}}{l} \right]; \quad (29)$$

$$\frac{C_{i,j} - C_{i-1,j}}{h} = \frac{1}{\mathrm{Sc}} \frac{1}{l} \times$$
$$\times \left[\frac{C_{i,j+1} - C_{i,j}}{l} \Lambda_{i,j+1} - \Lambda_{i,j} \frac{C_{i,j} - C_{i,j-1}}{l} \right], \quad (30)$$

where

$$\Lambda_{i,j} = \frac{(\rho'_{i,j} + \rho'_{i,j-1})^2}{4} \frac{u'_{i,j} + u'_{i,j-1}}{2}.$$

Conditions (16) will be expressed also in finite-difference form,

$$\left(\frac{\partial Z}{\partial \psi'} \right)_{i,0} \approx \frac{Z_{i,1} - Z_{i,0}}{l} = 0;$$
$$\left(\frac{\partial Z}{\partial \psi'} \right)_{i,n} \approx \frac{Z_{i,n-1} - Z_{i,n}}{l} = 0, \quad (31)$$

where Z = u', H', C.

An iterative technique is applied to the solution of the obtained system of nonlinear equations. To obtain the iterations $u'^s_{i,j}$, system (28) is written in the form

$$\frac{h}{l^2} \Lambda^{s-1}_{i,j+1} u'^s_{i,j+1} - \left[1 + \frac{h}{l^2} \left(\Lambda^{s-1}_{i,j+1} + \Lambda^{s-1}_{i,j} \right) \right] u'^s_{i,j} +$$
$$+ \frac{h}{l^2} \Lambda^{s-1}_{i,j-1} = \frac{p'^t_i - p'_{i-1}}{\rho'^s_{i,j} u'^s_{i,j}} - u'_{i-1,j}. \quad (32)$$

Systems (29) and (30) can be written in a similar manner. First, we fix the pressure, assuming that $p'^t_i = p'_{i-1}$. As the zero approximation for $\rho'^{s-1}_{i,j}$, $u'^{s-1}_{i,j}$, we take their values from the preceding layer, and then evaluate the first approximation for all the values to be determined (for fixed pressure). For evaluating the second approximation, we assume $u'^{s-1}_{i,j} = (u'^0_{i,j} + u'^1_{i,j})/2$; $\rho'^{s-1}_{i,j} = (\rho'^0_{i,j} + \rho'^1_{i,j})/2$, and so forth. Solutions to (32) and similar equations for determining $H'_{i,j}$, $C_{i,j}$ for conditions (16) and (31) were obtained by a brute force technique [7]. The conditions for which the iteration process ceased to converge have the form

$$\max |u'^s_{i,j} - u'^{s-1}_{i,j}| < \varepsilon_1; \quad \max |\rho'^s_{i,j} - \rho'^{s-1}_{i,j}| < \varepsilon_2. \quad (33)$$

After fulfilling conditions (33) with respect to the obtained values of ρ' and u', we checked the inequality

$$\left| \frac{1 + \rho'_- u'_-}{2} \int_0^1 \frac{d\psi'}{\rho' u'} - 1 \right| < \varepsilon_3, \quad (34)$$

which follows from conditions (5) for mass conservation and the first relation in system (12). The selected value for the pressure was considered correct if (34) was fulfilled. Otherwise, the value of $p_1'^t$ was changed, depending on the value of the integral $\int_0^1 \frac{d\psi'}{\rho' u'}$. The calculations were then repeated until conditions (33) and (34) were fulfilled. When this was accomplished, we turned to the next layer, and so forth.

Conditions (6) for total momentum conservation and for conservation of the enthalpies (7) were used to check the obtained solutions. Computations showed that these conditions were fulfilled with the same degree of accuracy as relation (34). Hence, in the plane (ξ', ψ'), the problem can be followed through to the end.

Transition to the physical plane (x', y') was achieved with the aid of the following formulas:

$$y' = \frac{1+\rho'_- u'_-}{2} \int_0^{\psi'} \frac{d\psi'}{\rho' u'}, \qquad (35)$$

$$x' = \frac{(1+\rho'_- u'_-)^2}{4\varkappa} \int_0^{\xi'} \frac{d\xi'}{b'(\xi')\,[u'_d(\xi') - u'_b(\xi')]}, \qquad (36)$$

which derive from the first relations in (12) and (13), respectively.

On the basis of the scheme developed above, burning of hydrogen in air was calculated on a 2M high-speed computer for Pr = Sc = 0.5. The conditions at the outlet are as follows: $u_+ = 70$ m/sec, $u'_- = 0.534$; $T_+ = 1000°$ K, $T'_- = 1$; $p_+ = p_0 = 1$ atm, $p_- = p_0 = 1$ atm; $(c_1)_+ = 0.2$, $(c_2)_- = 0.05$; $(c_4)_+ = 0.8$, $(c_4)_- = 0.95$; $(c_3)_+ = 0$, $(c_3)_- = 0$. The excess oxidant ratio is $\alpha = 1.58$.

Figure 2 shows the profiles of 1) oxidizer concentration, 2) fuel concentration, 3) product concentration, and 4) temperature, in two cross sections, together with the flame-front shape 5). From the figure, it can be seen that the flame front closes at the axis of the fuel jet (according to expectations, because of the condition $\alpha > 1$), while at the surface of the flame front, the product concentration and mixture temperature have maximum constant values.

Figure 3 shows velocity and density profiles which reveal that for parallel-jet mixing and diffusion burning, the velocity profiles can possess a maximum not at the boundaries of but inside the mixing zone. This may be attributed to the low density of the mixture in the zone adjacent to the flame front and to the action of a favorable pressure gradient.

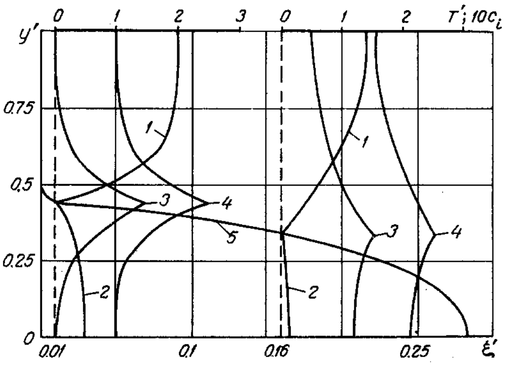

Fig. 2. Profiles of 1) oxidizer concentration, 2) fuel concentration, 3) product concentration, 4) temperature, in two cross sections of the jet ($\xi' = 0.01$; $\xi' = 0.16$); 5) shape of flame front.

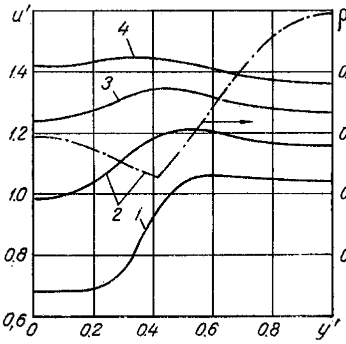

Fig. 3. Velocity profiles in the cross sections $\xi' = 0.01$ (1), $\xi' = 0.05$ (2), $\xi' = 0.1$ (3), and the density profile for $\xi' = 0.05$.

Figure 4 shows (1) the relation between the variables $\varkappa x'$ and ξ', (2) the changes in velocities $u'(\xi', 1)$, (3) $u'(\xi', 0)$, (4) the maximum velocity u'_d, as well as (5) the changes in the pressure difference along the axis. The phenomenon we have pointed out resembles in many ways the known phenomenon in tubes [8, 9], where in the case of external heat input to the tube wall, a subsonic flow within the tube is accelerated while the pressure along the tube decreases.

In the calculations for point $\xi' = 0$, $\psi' = (\rho'_- u'_-)/(1+\rho'_- u'_-)$, it was assumed that $u' = 1 + u'_-)/2$, $\rho' = (1 + \rho'_-)/2$, $H' = (1 + H'_-)/2$, and C = 0.5; in formula (36), we have $b'(0) = 2l$ when passing to the plane x', y'. The values employed for ε_i were $\varepsilon_1 = \varepsilon_2 = 0.001$, and $\varepsilon_3 = 0.01$.

NOTATION

x, y and u, v are the longitudinal and transverse coordinates and velocities, respectively; p is pressure; m is the molecular weight; c_p is the specific heat at constant pressure; H is the total enthalpy of mixture;

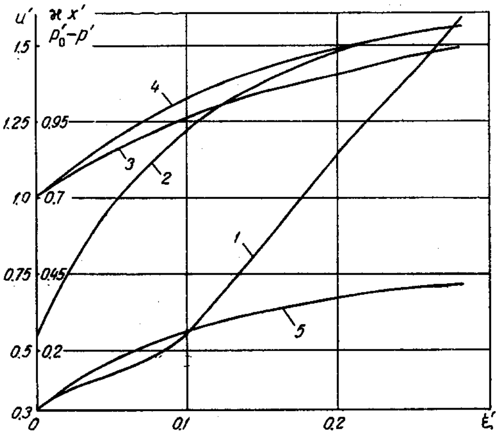

Fig. 4. (1) relation between the variables $\varkappa x'$ and ξ', (2) behavior of velocity $u'(\xi', 1)$, (3) behavior of velocity $u'(\xi', 0)$, (4) behavior of maximum velocity $u'_d(\xi')$, and (5) changes in the pressure difference along the axis.

ε is the kinematic coefficient of turbulent viscosity; Pr and Sc are the turbulent Prandtl and Schmidt numbers; c_i, w_i, h_i^*, and c_{pi}, are the mass concentration, mass-formation rate, heat of formation, specific heat at constant pressure of the i-th component, m_i is the molecular weight of the i-th component, respectively; R_0 is the universal gas constant; T is the absolute temperature; N is the number of components; p_0 is the initial pressure; $\varkappa$ is the empirical turbulence constant; b(x) is the width of mixing zone; α is the excess oxidizer ratio; x_i is the abscissa of initial cross section of main zone. Subscripts: d is for the maximum value of a function; b is for the minimum value of a function; (+) is for the initial value for nozzle 1; (-) is for the initial value for nozzle 2; $(*)$ is for the value of a function at the flame front. A bar denotes dimensionless values.

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